

Supersymmetric D-branes in the D1-D5 background

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ABSTRACT: We construct supersymmetric D-brane probe solutions in the background of the 2-charge D1-D5 system on M , where M is either $K3$ or T^4 . We focus on ‘near-horizon bound states’ that preserve supersymmetries of the near-horizon $AdS_3 \times S^3 \times M$ geometry and are static with respect to the global time coordinate. We find a variety of half-BPS solutions that span an AdS_2 subspace in AdS_3 , carry worldvolume flux and can wrap an S^2 within S^3 and/or supersymmetric cycles in M .

KEYWORDS: D-branes, Black Holes in String Theory, AdS-CFT Correspondence.

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1. Introduction and summary

The study of D-brane probes in the near-horizon region of BPS D-brane systems has recently had interesting applications to the counting of BPS degeneracies. D-brane systems with an $AdS_p \times S^q$ near-horizon region where supersymmetry is enhanced allow for D-brane probe configurations localized near the horizon preserving a portion of the enhanced supersymmetries. Such branes have been constructed in backgrounds with $AdS_2 \times S^2$ [1], $AdS_3 \times S^2$ [2] and $AdS_2 \times S^3$ [3] near-horizon geometries. The branes considered in these papers possess a number of interesting properties. The solutions of interest are static with respect to a choice of global time coordinate, and supersymmetry fixes their radial position in AdS in terms of their charges. Furthermore, they preserve half of the near-horizon supersymmetries but break all of the supersymmetries of the full asymptotically flat geometry. In case they wrap the sphere or a cycle in the internal space with worldvolume flux turned on, they carry lower D-brane charge and can be seen as bound states of lower-dimensional D-branes through a form of the Myers effect [4, 5].

It is natural to interpret such branes as ‘near-horizon bound states’ of the D-brane system, and one would expect that by quantizing their moduli one should be able to count degeneracies of BPS states. This expectation was borne out for the D0-D4 black hole in type IIA, where the quantum mechanical counting reduces to counting lowest Landau levels in a magnetic field on the internal space and reproduces the entropy both for the ‘large’ [6] and ‘small’ [7] black hole cases. Furthermore, for black holes constructed out of M5-branes, the elliptic genus can be reconstructed by counting near-horizon wrapped membrane states [2, 8].

Motivated by these results, we revisit the two-charge D1-D5 system on M (where M can be T^4 or K_3), forming a black string in 6 dimensions (see [9, 10] for reviews). This system has a large ground-state degeneracy, the logarithm of which is proportional to $\sqrt{Q_1 Q_5}$ for large charges. We will look for supersymmetric D-branes in the near horizon $AdS_3 \times S^3 \times M$ region. In earlier works, half-BPS solutions which carry momentum along certain directions, such as giant gravitons [11] and branes wrapping S^3 with momentum along AdS_3 [12] were constructed. The branes we will consider here differ from these in that they do not carry any momentum and are entirely static with respect to global time. We allow the branes to carry arbitrary worldvolume fluxes (and hence also induced lower-dimensional D-brane charges).

The 16 supersymmetries of the near-horizon region split into 8 supersymmetries that extend to the full asymptotically flat solution, which we will call ‘Poincaré supersymmetries’, and 8 ‘enhanced’ supersymmetries that exist only in the near-horizon limit. They are most easily distinguished in Poincaré coordinates in AdS_3 . D-brane probes preserving some Poincaré supersymmetries should have a BPS counterpart in the full geometry, and we will verify for each solution whether it preserves Poincaré supersymmetries.

The outcome of our classification yields a large variety of D-branes preserving half of the near-horizon supersymmetries and is summarized in the following table.

| brane type | AdS_3 | S^3 | M | near-horizon susy | Poincaré susy |
|------------|---------|---------|---------|-------------------|---------------|
| D1 | AdS_2 | \cdot | \cdot | 1/2 | 1/2 |
| D3 | AdS_2 | \cdot | 2-cycle | 1/2 | 1/2 |
| D5 | AdS_2 | \cdot | M | 1/2 | 1/2 |
| D3 | AdS_2 | S^2 | \cdot | 1/2 | 1/2 |
| D7 | AdS_2 | S^2 | M | 1/2 | 1/2 |

The solutions come in two types: branes of the first type span an AdS_2 subspace in $AdS_3 \times S^3$ (and possibly wrap a supersymmetric cycle in M) while the second type of branes spans an $AdS_2 \times S^2$ subspace in $AdS_3 \times S^3$ (and possibly wrap the whole of M). Branes of the second type are dipolar as the S^2 is contractible within S^3 , and are stabilized by worldvolume flux [13, 14]. The size of the S^2 is quantized in terms of the number of fundamental strings bound to the D-brane. In all the above solutions, the radial position in AdS_3 is fixed in terms of the charges and it is natural to view them as ‘near-horizon bound states’ of the D1-D5 system. One novel feature is that, contrary to the examples in other backgrounds discussed above, these probe branes do preserve half of the Poincaré supersymmetries of the full asymptotically flat geometry.

Let us comment on related D-brane solutions that have appeared in the literature. The $AdS_2 \times S^2$ branes were studied from the point of view of the DBI action in [14, 15]. There is a substantial body of work discussing D-branes in the S-dual F1-NS5 background starting with [16]. The S-dual versions of branes with worldvolumes AdS_2 and $AdS_2 \times S^2$ appear there (the latter was shown to be half-BPS). A sampling of further studies of AdS_2 branes in the NS background includes [17]. We also want to point out that D-branes with an AdS_2 component to their worldvolume are known to exist in other D-brane backgrounds as well [18–21].

This paper is organized as follows. In section 2 we construct the Killing spinors on $AdS_3 \times S^3 \times M$ in suitable coordinates and review the conditions for probe branes to preserve supersymmetry. In section 3 we construct supersymmetric branes which extend along AdS_2 and possibly wrap cycles on M . In section 4 we turn to branes that span $AdS_2 \times S^2$ and possibly wrap cycles on M . We end with a discussion of open problems in 5. Appendix A discusses how Poincaré supersymmetries extend to the full geometry and appendix B discusses some branes spanning other submanifolds, none of which were found to be supersymmetric.

2. The near-horizon limit of the D1-D5 system

In this section, we review some properties of the near-horizon limit of the D1-D5 system needed in the rest of the paper.

2.1 Background

Consider type IIB on $S^1 \times M$ (with M being either K_3 or T^4), with D5-branes wrapped on $S^1 \times M$ and D1-branes on S^1 . We will take the S^1 radius to infinity in what follows, so that the configuration looks like a black string in six dimensions. The near-horizon supergravity

background is $AdS_3 \times S^3 \times M$ with constant dilaton and nonvanishing RR 3-form flux $F^{(3)} = dC^{(2)}$. On AdS_3 , we will use a global ‘anti-de Sitter’ coordinate system (τ, ω, ξ) in which the constant ξ slices are isomorphic to AdS_2 . The supergravity background is then given by (see e.g. [24]):

$$\begin{aligned}
 ds^2 &= r_1 r_5 [d\xi^2 + \cosh^2 \xi (-\cosh^2 \omega d\tau^2 + d\omega^2) \\
 &\quad + d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\varphi^2)] + \frac{r_1}{r_5} ds_M^2 \\
 e^{-\phi} &= \frac{1}{g} \frac{r_5}{r_1} \\
 C^{(2)} &= \frac{r_5^2}{g} \left[\left(\xi + \frac{1}{2} \sinh 2\xi \right) \cosh \omega d\omega \wedge d\tau + \left(\psi - \frac{1}{2} \sin 2\psi \right) \sin \theta d\theta \wedge d\varphi \right] \quad (2.1)
 \end{aligned}$$

where ds_M^2 is a Ricci-flat metric on M and

$$\begin{aligned}
 r_5 &= \sqrt{g Q_5 \alpha'} \\
 r_1 &= \frac{4\pi^2 \alpha'}{\sqrt{V_M}} \sqrt{g Q_1 \alpha'} \quad (2.2)
 \end{aligned}$$

with V_M the volume of M in the metric ds_M^2 and Q_1, Q_5 the D1- and D5 charges. The coordinates τ, ω, ξ vary over \mathbf{R} while $0 \leq \psi, \theta < \pi, 0 \leq \varphi < 2\pi$.

2.2 Killing spinors

The D1-D5 background preserves 8 supersymmetries which get enhanced to 16 supersymmetries in the near-horizon limit. We will now derive the explicit expression for the near-horizon Killing spinors needed in the following sections.

In type IIB supergravity, the supersymmetry variation parameter ε consists of two chiral spinors of the same chirality:

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \quad (2.3)$$

where $\Gamma_{(10)} \varepsilon_{1,2} = \varepsilon_{1,2}$ with $\Gamma_{(10)} \equiv \Gamma^0 \dots \Gamma^9$. We now examine the conditions for ε to be a Killing spinor. Our supergravity conventions follow [25] and the dilatino and gravitino variations read¹

$$\begin{aligned}
 \delta\lambda &= -\frac{1}{4} e^\phi \mathcal{F}^{(3)} \sigma^1 \varepsilon \\
 \delta\Psi_{\hat{M}} &= \nabla_{\hat{M}} \varepsilon + \frac{e^\phi}{8} \mathcal{F}^{(3)} \Gamma_{\hat{M}} \sigma^1 \varepsilon
 \end{aligned}$$

The vanishing of the dilatino variation amounts to a chirality projection

$$\Gamma_{(6)} \varepsilon = \Gamma_{(4)} \varepsilon = -\varepsilon \quad (2.4)$$

¹Our 10D index conventions are as follows: $M, N = 0, \dots, 9, \mu, \nu = 0, \dots, 5, m, n = 6, \dots, 9$. Hatted indices $\hat{M}, \hat{\mu}, \hat{m}$ refer to a coordinate basis while unhatted ones M, μ, m are orthonormal frame indices.

where $\Gamma_{(6)} \equiv \Gamma^0 \dots \Gamma^5$, $\Gamma_{(4)} \equiv \Gamma^6 \dots \Gamma^9$. while the gravitino variation with index on the internal manifold M imposes that ε is covariantly constant in the internal directions:

$$\nabla_{\hat{m}} \varepsilon = 0. \tag{2.5}$$

The gravitino variation with index on $AdS_3 \times S^3$ then leads to the equations

$$\delta \Psi_{\hat{\mu}} = \left[\nabla_{\hat{\mu}} + \frac{1}{2\sqrt{r_1 r_5}} \Gamma^{012} \Gamma_{\hat{\mu}} \sigma^1 \right] \varepsilon = 0 \tag{2.6}$$

Symmetry dictates that solutions to this equation should be given by multiplying a constant spinor by an $SL(2, R) \times SU(2)$ group element in a suitable representation [26]. Expressing the spin connection on $AdS_3 \times S^3$ in terms of the following vielbein²

$$\begin{aligned} e^0 &= \sqrt{r_1 r_5} \cosh \xi \cosh \omega dt & e^3 &= \sqrt{r_1 r_5} d\psi \\ e^1 &= \sqrt{r_1 r_5} \cosh \xi d\omega & e^4 &= \sqrt{r_1 r_5} \sin \psi d\theta \\ e^2 &= \sqrt{r_1 r_5} d\xi & e^5 &= \sqrt{r_1 r_5} \sin \psi \sin \theta d\phi. \end{aligned}$$

one finds the solutions

$$\varepsilon = e^{\frac{\xi}{2} \Gamma^{02} \sigma^1} e^{\frac{\omega}{2} \Gamma^{10} \sigma^1} e^{\frac{\tau}{2} \Gamma^{21} \sigma^1} e^{\frac{\pi-\psi}{2} \Gamma^{45} \sigma^1} e^{\frac{\pi-\theta}{2} \Gamma^{35} \sigma^1} e^{\frac{\phi}{2} \Gamma^{43} \sigma^1} \varepsilon_0 \tag{2.7}$$

Here, ε is independent of the $AdS_3 \times S^3$ coordinates and satisfies the conditions (2.4), (2.5):

$$\begin{aligned} \partial_{\hat{\mu}} \varepsilon_0 &= \nabla_{\hat{m}} \varepsilon_0 = 0 \\ \Gamma_{(6)} \varepsilon_0 &= \Gamma_{(4)} \varepsilon_0 = -\varepsilon_0 \end{aligned} \tag{2.8}$$

We can write a more explicit expression for ε_0 by decomposing the $SO(1, 9)$ gamma matrices under the $SO(1, 5) \times SO(4)$ subgroup as follows:

$$\Gamma^\mu = \gamma^\mu \otimes 1 \quad \mu = 0 \dots 5 \tag{2.9}$$

$$\Gamma^m = \gamma_{(6)} \otimes \gamma^m \quad m = 6 \dots 9 \tag{2.10}$$

where γ^μ and γ^m are $SO(1, 5)$ and $SO(4)$ gamma matrices respectively, and we have defined $\gamma_{(6)} = \gamma^0 \dots \gamma^5$, $\gamma_{(4)} = \gamma^6 \dots \gamma^9$. The ten-dimensional chirality operator is $\Gamma_{(10)} \equiv \Gamma^0 \dots \Gamma^9 = \gamma_{(6)} \otimes \gamma_{(4)}$. A chiral spinor in ten dimensions then decomposes as

$$\mathbf{16} \rightarrow (\mathbf{4}, \mathbf{2}) + (\mathbf{4}', \mathbf{2}').$$

where the unprimed (primed) representations have positive (negative) chirality. When M is K_3 , we take the convention that the representation $\mathbf{2}$ forms a doublet under the $SU(2)$ holonomy, while $\mathbf{2}'$ consists of two holonomy singlets. The chirality condition in (2.8)

²Note that this is not the left-invariant basis on the $SL(2, R) \times SU(2)$ group manifold but rather a linear combination of left- and right invariant forms.

projects out the $(\mathbf{4}, \mathbf{2})$ component. Choosing basis elements η_+, η_- for the covariantly constant $\mathbf{2}'$ spinors, we can take the following ansatz for ε_0 :

$$\varepsilon_0 = \begin{pmatrix} \epsilon_1^+ \\ \epsilon_2^+ \end{pmatrix} \otimes \eta_+ + \begin{pmatrix} \epsilon_1^- \\ \epsilon_2^- \end{pmatrix} \otimes \eta_- \tag{2.11}$$

with ϵ^\pm constant and antichiral ($\gamma_{(6)}\epsilon^\pm = -\epsilon^\pm$) doublets on $AdS_3 \times S^3$ and η_\pm covariantly constant and antichiral ($\gamma_{(4)}\eta_\pm = -\eta_\pm$) spinors on M . Both for $M = T^4$ and $M = K3$, we have 16 independent Killing spinors.

2.3 Poincaré supersymmetries

The following coordinate transformation takes us to Poincaré coordinates (t, x, u) for AdS_3 :

$$\begin{aligned} u &= \frac{1}{\sqrt{r_1 r_5}} (\cosh \xi \cosh \omega \cos \tau + \cosh \xi \sinh \omega) \\ t &= \frac{1}{u} (\cosh \xi \cosh \omega \sin \tau) \\ x &= \frac{1}{u} \sinh \xi. \end{aligned} \tag{2.12}$$

The AdS_3 part of the metric and 3-form become

$$\begin{aligned} ds_{AdS_3}^2 &= r_1 r_5 \left[u^2 (-dt^2 + dx^2) + \frac{du^2}{u^2} \right] \\ F_{AdS_3}^{(3)} &= \frac{2r_5^2}{g} u dt \wedge dx \wedge du. \end{aligned}$$

The 16 near-horizon Killing spinors split into 8 spinors that extend to the full asymptotically flat spacetime (as they generate a Poincaré superalgebra we will henceforth refer to them as ‘Poincaré supersymmetries’) and 8 spinors corresponding to enhanced near-horizon supersymmetries (generating special conformal transformations). In Poincaré coordinates, the Poincaré supersymmetries are time-independent and are given by:

$$\varepsilon_P = \sqrt{u} R \varepsilon_- \tag{2.13}$$

where R is the $SU(2)$ group element

$$R = e^{\frac{\psi-\theta}{2} \Gamma^{45} \sigma^1} e^{\frac{\psi-\theta}{2} \Gamma^{35} \sigma^1} e^{\frac{\phi}{2} \Gamma^{43} \sigma^1}$$

and ε_- is a spinor that satisfies, in addition to (2.8), the extra projection condition

$$\Gamma^{01} \sigma_1 \varepsilon_- = -\varepsilon_- \tag{2.14}$$

Here we have numbered the coordinates as $(x^0, x^1, x^2) = (t, x, u)$. See appendix A for more details on how the Poincaré supersymmetries extend to the full asymptotically flat geometry.

2.4 Supersymmetric D-brane probes

A supersymmetry of the background is preserved in the presence of a bosonic Dp-brane configuration if it can be compensated for by a κ -symmetry transformation [27]. This can be expressed as a projection equation

$$(1 - \Gamma)\varepsilon = 0 \tag{2.15}$$

where Γ (satisfying $\text{tr}\Gamma = 0$, $\Gamma^2 = 1$) is the operator entering in the κ -symmetry transformation rule on the Dp-brane and ε is a general Killing spinor (constructed above) pulled back to the world-volume. The operator Γ can be written in a simple form in a special worldvolume Lorentz frame in which the worldvolume field strength F takes the form

$$2\pi\alpha'F = \tanh\Phi_0 e^{\underline{0}} \wedge e^{\underline{1}} + \sum_{r=1}^{(p-1)/2} \tan\Phi_r e^{2r} \wedge e^{2r+1}.$$

Γ is then given by [27]

$$\Gamma = e^{-a}\Gamma_{(0)}(\sigma_3)^{\frac{p-3}{2}}i\sigma_2 \tag{2.16}$$

with

$$\begin{aligned} \Gamma_{(0)} &= \Gamma_{\underline{0}\dots\underline{p}} \\ a &= \sum_{r=0}^{(p-1)/2} \Phi_r \Gamma^{2r, 2r+1} \sigma_3. \end{aligned} \tag{2.17}$$

In the above formulas, underlined indices are orthonormal frame indices on the D-brane worldvolume.

3. D-branes spanning AdS_2

In this section, we will consider D-branes that span an AdS_2 subspace within $AdS_3 \times S^3$. They can be taken to be embedded at constant $\xi = \xi_0$ in the coordinates (2.1). We will see that the requirement of supersymmetry fixes ξ_0 in terms of the charges carried by the brane.

3.1 D1-brane along AdS_2

3.1.1 Near-horizon supersymmetries

We consider a D1-brane probe embedded in AdS_3 at constant $\xi = \xi_0$ and static in the remaining $S^3 \times M$ directions. The worldvolume coordinates can be taken to be τ, ω . We allow for an electric field on the worldvolume which is conveniently parametrized as

$$2\pi\alpha'F = \tanh\Phi_0 e^0 \wedge e^2.$$

Here and in what follows, with a slight abuse of notation, the e^a stand for the corresponding target space vielbein elements pulled-back to the world-volume. Explicitly, we have

$$\begin{aligned} e^0 &= \sqrt{r_1 r_5} \cosh\xi_0 \cosh\omega d\tau \\ e^2 &= \sqrt{r_1 r_5} \cosh\xi_0 d\omega \end{aligned} \tag{3.1}$$

The supersymmetries preserved by the brane should satisfy $(1 - \Gamma)\varepsilon = 0$ with

$$\Gamma = e^{-\Phi_0 \Gamma^{02} \sigma^3} \Gamma_{02} \sigma^1$$

$$\varepsilon = e^{\frac{\xi_0}{2} \Gamma^{20} \sigma^1} e^{\frac{\omega}{2} \Gamma^{10} \sigma^1} e^{\frac{\tau}{2} \Gamma^{21} \sigma^1} R_0 \varepsilon_0$$

where R_0 is a constant $SU(2)$ group element depending on the position in the S^3 given by

$$R_0 \equiv R(\psi_0, \theta_0, \phi_0) = e^{\frac{\psi_0 - \theta_0}{2} \Gamma^{45} \sigma^1} e^{\frac{\psi_0 + \theta_0}{2} \Gamma^{35} \sigma^1} e^{\frac{\phi_0}{2} \Gamma^{43} \sigma^1}.$$

Imposing $(1 - \Gamma)\varepsilon = 0$ for all values of τ, ω leads to two equations

$$(1 - s e^{-a} \Gamma_{01} \sigma^1) e^{s \frac{\xi_0}{2} \Gamma^{02} \sigma^1} R_0 \varepsilon_0 = 0 \quad (3.2)$$

where $s = \pm 1$. Multiplying with $e^{s \frac{\xi_0}{2} \Gamma^{02} \sigma^1}$ and taking linear combinations one finds that a solution exists if

$$\tanh \xi_0 = -\frac{1}{\cosh \Phi_0} \quad \Leftrightarrow \quad |\tanh \Phi_0| = \frac{1}{\cosh \xi_0}.$$

Plugging in our ansatz for ε_0 (2.11), the projection condition on the surviving supersymmetries can be written in terms of the 6-dimensional spinor doublet ϵ^\pm as

$$(1 - \text{sgn}(\Phi_0) R_0^{-1} \gamma_{02} \sigma^3 R_0) \epsilon^\pm = 0. \quad (3.3)$$

Hence the brane preserves half the supersymmetries of the background, and the preserved supercharges depend on the position of the brane on S^3 through R_0 as well as on the sign of Φ_0 .

The latter is related to the sign of the fundamental string charge bound to the D1-brane. Indeed, for nonzero Φ_0 , the D1-brane acts as a source for the B-field and carries an induced fundamental string charge as well. Demanding that it is properly quantized and equal to q imposes a quantization condition on Φ_0 :

$$\sinh \Phi_0 = \frac{g r_1}{r_5} q.$$

Note that, from (3.3), it follows that branes carrying opposite fundamental string charge can preserve the same supersymmetries provided they sit at antipodal locations on the S^3 . A similar property was observed for branes in other $AdS_p \times S^q$ backgrounds [1, 2].

The radial position ξ_0 is determined by the fundamental string charge as

$$\sinh \xi_0 = \frac{r_5}{g r_1} \frac{1}{|q|}.$$

Of course, the above equations provide a solution to the equations of motion following from the DBI action as one can easily verify.

The above $(q, 1)$ string solution S-dualizes to a $(1, q)$ string in the F1-NS5 background. For $q = 1$, the latter solution was found from the DBI equations of motion in [16]. Our analysis implies that this solution should be supersymmetric as well.

3.1.2 Poincaré supersymmetries

We now check whether the above solution preserves any Poincaré supersymmetries. A D1-brane at constant $\xi = \xi_0$ satisfies, in Poincaré coordinates (2.12),

$$u(x) = \frac{\sinh \xi_0}{x}.$$

Taking (t, x) to be the worldvolume coordinates, the κ -projector becomes

$$\Gamma = e^{-\Phi_0 \Gamma_{01} \sigma_3} \Gamma_{01} \sigma_1$$

with

$$\Gamma_{01} = -\tanh \xi_0 \Gamma_{01} + \frac{1}{\cosh \xi_0} \Gamma_{02}. \quad (3.4)$$

To check whether the D1-brane preserves some fraction of the Poincaré supersymmetries, we need to verify whether the equation $(1 - \Gamma)\varepsilon_P$ (with ε_P given in (2.13)) has any solutions. Using (2.14) one finds the equation

$$\left[1 + \cosh \Phi_0 \tanh \xi_0 - \frac{\cosh \Phi_0}{\cosh \xi_0} \Gamma_{02} \sigma_1 + \sinh \Phi_0 i \sigma_2 \right] R \varepsilon_- = 0$$

As before, a solution exists when $\tanh \xi_0 = -\frac{1}{\cosh \Phi_0}$ and requires

$$(1 \pm R_0^{-1} \Gamma_{02} \sigma_3 R_0) \varepsilon_- = 0.$$

where the sign again depends on the sign of Φ_0 . This projection condition is compatible with (2.14) and we conclude that the D1-brane preserves half of the Poincaré supersymmetries.

3.2 D3-branes along AdS_2 and wrapping a 2-cycle in M

3.2.1 Near-horizon supersymmetries

Here, we consider a D3-brane spanning and AdS_2 subspace in AdS_3 at $\xi = \xi_0$ and wrapping a 2-cycle Σ in M . We denote the pull-back of the induced volume form on Σ by vol_Σ and define a corresponding Γ -matrix combination:

$$\Gamma_\Sigma = \frac{1}{2\sqrt{g'}} \epsilon^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}} \quad (3.5)$$

with $g'_{\hat{a}\hat{b}}$ the induced metric on Σ . We parametrize the worldvolume flux as

$$2\pi\alpha' F = \tanh \Phi_0 e^0 \wedge e^2 + \tan \Phi_1 \text{vol}_\Sigma$$

and take $\cos \Phi_1 \geq 0$. The κ -projector is given by

$$\Gamma = e^{-\Phi_0 \Gamma^{02} \sigma_3} e^{-\Phi_1 \Gamma_\Sigma \sigma_3} \Gamma_{02} \Gamma_\Sigma i \sigma_2.$$

Imposing the supersymmetry condition $(1 - \Gamma)\varepsilon = 0$ leads to two equations

$$(1 - e^{-\Phi_0 \Gamma^{02} \sigma_3} e^{-s\Phi_1 \Gamma_\Sigma \sigma_3} \Gamma_{02} \Gamma_\Sigma i \sigma_2) e^{s\frac{\xi_0}{2} \Gamma^{02} \sigma_1} R_0 \varepsilon_0 = 0$$

with $s = \pm 1$ and R_0 defined in (3.2). After some manipulations one finds that a solution exists if

$$\tanh \xi_0 = -\frac{\sin \Phi_1}{\cosh \Phi_0} \quad (3.6)$$

and requires the projection

$$\left(1 - R_0^{-1} \left(\frac{\sinh \Phi_0}{\cos \Phi_1} \Gamma_\Sigma \sigma_1 + \frac{\cosh \Phi_0}{\cosh \xi_0 \cos \Phi_1} \Gamma_{02} \Gamma_\Sigma i \sigma_2 \right) R_0 \right) \varepsilon_0 = 0$$

This projection equation can be rewritten in a more standard form using the identity

$$\frac{\cosh \Phi_0}{\cosh \xi_0} = \sqrt{\sinh^2 \Phi_0 + \sin^2 \Phi_1}$$

which follows from (3.6). The projection condition becomes

$$\left(1 + R_0^{-1} e^{-\frac{\alpha}{2} \Gamma^{02} \sigma_3} \Gamma_{02} \Gamma_\Sigma i \sigma_2 e^{\frac{\alpha}{2} \Gamma^{02} \sigma_3} R_0 \right) \varepsilon_0 = 0 \quad (3.7)$$

with α defined by

$$\sinh \alpha = \frac{\sinh \Phi_0}{\cos \Phi_1}.$$

In the above equation, both Γ_Σ and ε_0 are in general dependent on the position on Σ and it is not trivial that the equation can be satisfied everywhere. This will be possible if Σ is a supersymmetric cycle. We proceed by plugging in our ansatz for ε_0 (2.11):

$$\varepsilon_0 = \begin{pmatrix} \epsilon_1^+ \\ \epsilon_2^+ \end{pmatrix} \otimes \eta_+ + \begin{pmatrix} \epsilon_1^- \\ \epsilon_2^- \end{pmatrix} \otimes \eta_- \quad (3.8)$$

In this ansatz, we are free to choose a convenient basis η_+, η_- for the internal covariantly constant spinors. It turns out that they can be chosen to be eigenstates of Γ_Σ . Indeed, when $M = T^4$, Σ is a T^2 within T^4 and Γ_Σ is position independent in suitable coordinates. The constant spinors η_+, η_- can be chosen to diagonalise Γ_Σ : $\Gamma_\Sigma \eta_\pm = \mp i \eta_\pm$.

When Σ is a supersymmetric cycle in $M = K3$ and, we can also choose η_+, η_- to diagonalise Γ_Σ . Because $K3$ is hyperkähler, it admits an S^2 family of complex structures, and we assume Σ to be holomorphic with respect to one of these complex structures. Choosing holomorphic coordinates $z^i, \bar{z}^{\bar{i}}$ with respect to this particular complex structure we can choose a basis η_+, η_- of covariantly constant spinors on $K3$ satisfying

$$\begin{aligned} \gamma_{\bar{i}} \eta_+ &= 0, & \gamma_{ij} \eta_+ &= \Omega_{ij} \eta_- \\ \gamma_i \eta_- &= 0, & \gamma_{\bar{i}\bar{j}} \eta_- &= -\bar{\Omega}_{\bar{i}\bar{j}} \eta_+. \end{aligned} \quad (3.9)$$

with Ω the (2, 0) form. Γ_Σ acts on η_\pm as $\Gamma_\Sigma \eta_\pm = \mp i \eta_\pm$.

Summarizing, both on $M = T^4$ and $M = K3$ we can take η_+, η_- to satisfy

$$\Gamma_\Sigma \eta_\pm = \mp i \eta_\pm.$$

Substituting into (3.7) gives projection conditions on the 6-dimensional spinor doublets

$$\epsilon^+ = \begin{pmatrix} \epsilon_1^+ \\ \epsilon_2^+ \end{pmatrix}, \quad \epsilon^- = \begin{pmatrix} \epsilon_1^- \\ \epsilon_2^- \end{pmatrix} : \quad \left(1 \pm R_0^{-1} e^{-\frac{\alpha}{2} \gamma^{02} \sigma_3} \gamma_{02} \sigma_2 e^{\frac{\alpha}{2} \gamma^{02} \sigma_3} R_0 \right) \epsilon^\pm = 0$$

We see that indeed half of the supersymmetries is preserved.

3.2.2 Poincaré supersymmetries

It's straightforward to show that these branes preserve half of the Poincaré supersymmetries as well. The projection condition on the Poincaré Killing spinors becomes

$$\left(1 - e^{-\Phi_0 \Gamma_{01} \sigma_3} e^{-\Phi_1 \Gamma_{\Sigma} \sigma_3} \Gamma_{01} \Gamma_{\Sigma} i \sigma_2\right) \varepsilon_- = 0$$

with Γ_{01} defined in (3.4). Using the equation of motion (3.6) and (2.14) this reduces to the projection equation

$$\left(1 + R_0^{-1} e^{-\frac{\alpha}{2} \Gamma^{02} \sigma_3} \Gamma_{02} \Gamma_{\Sigma} i \sigma_2 e^{\frac{\alpha}{2} \Gamma^{02} \sigma_3} R_0\right) \varepsilon_- = 0$$

with α defined by

$$\sinh \alpha = \frac{\sinh \Phi_0}{\cos \Phi_1}.$$

Note that this projection condition is compatible with (2.14) and can be solved as in the previous section using the fact that Σ is a supersymmetric cycle. Hence we conclude that the D3-brane preserves half of the Poincaré supersymmetries.

3.3 D5-branes spanning $AdS_2 \times M$

3.3.1 Near-horizon supersymmetries

In this subsection we consider a D5-brane spanning and AdS_2 subspace in AdS_3 at $\xi = \xi_0$ and wrapping the whole of M . Choosing suitable complex coordinates on M we can take the the worldvolume flux as

$$2\pi\alpha' F = \tanh \Phi_0 e^0 \wedge e^2 + i \tan \Phi_1 e^1 \wedge e^{\bar{1}} + i \tan \Phi_2 e^2 \wedge e^{\bar{2}}$$

with $\cos \Phi_{1,2} \geq 0$. The κ -projector is given by

$$\Gamma = e^{-\Phi_0 \Gamma^{02} \sigma_3} e^{-i\Phi_1 \Gamma^{1\bar{1}} \sigma_3} e^{-i\Phi_2 \Gamma^{2\bar{2}} \sigma_3} \Gamma_{02} \Gamma_{(4)} \sigma_1.$$

Requiring $(1 - \Gamma)\varepsilon = 0$ and using the chirality property (2.8) leads to two equations

$$(1 + s e^{-\Phi_0 \Gamma^{02} \sigma_3} e^{-is(\Phi_1 + \Phi_2) \Gamma^{1\bar{1}} \sigma_3} \Gamma_{02} \sigma_1) e^{s \frac{\xi_0}{2} \Gamma^{02} \sigma_1} R_0 \varepsilon_0 = 0$$

with $s = \pm 1$. Note that only the sum $\Phi \equiv \Phi_1 + \Phi_2$ of the worldvolume flux parameters on M enters the equations, while their difference is left undetermined. After some algebra, one finds that a solution exists if

$$\tanh \xi_0 = \frac{\cos \Phi}{\cosh \Phi_0}. \tag{3.10}$$

and requires the projection

$$\left(1 + R_0^{-1} e^{-\frac{\alpha}{2} \Gamma^{02} \sigma_3} \Gamma_{02} \Gamma_{1\bar{1}} \sigma_2 e^{\frac{\alpha}{2} \Gamma^{02} \sigma_3} R_0\right) \varepsilon_0 = 0 \tag{3.11}$$

where we defined α by

$$\sinh \alpha = \frac{\sinh \Phi_0}{\sin \Phi}.$$

We proceed by plugging in the ansatz for ε_0 (2.11) and choosing internal spinors η_+ , η_- satisfying

$$\Gamma_{1\bar{1}}\eta_{\pm} = \mp\eta_{\pm}.$$

This can trivially be done for $M = T^4$, while for $M = K3$ one can choose η_{\pm} to obey (3.9). The resulting 6D projection conditions

$$\left(1 \pm R_0^{-1} e^{-\frac{\alpha}{2}\Gamma^{02}\sigma_3}\Gamma_{02}\Gamma_{1\bar{1}}\sigma_2 e^{\frac{\alpha}{2}\Gamma^{02}\sigma_3}R_0\right)\varepsilon^{\pm} = 0$$

show that the brane is half-BPS.

3.3.2 Poincaré supersymmetries

As in the previous cases, these branes preserve half of the Poincaré supersymmetries as well. The κ -projection condition on the Poincaré Killing spinors (2.13) leads to a single equation

$$\left(1 + e^{-\Phi_0}\Gamma_{0\bar{1}}\sigma_3 e^{-i\Phi}\Gamma^{1\bar{1}}\sigma_3\Gamma_{0\bar{1}}\sigma_1\right)\varepsilon_- = 0$$

with $\Gamma_{0\bar{1}}$ defined in (3.4). Using the equation of motion (3.10) and (2.14) this reduces to the projection equation

$$\left(1 + R_0^{-1} e^{-\frac{\alpha}{2}\Gamma^{02}\sigma_3}\Gamma_{02}\Gamma_{1\bar{1}}\sigma_2 e^{\frac{\alpha}{2}\Gamma^{02}\sigma_3}R_0\right)\varepsilon_- = 0$$

with α again defined by

$$\sinh \alpha = \frac{\sinh \Phi_0}{\sin \Phi}$$

This projection condition is compatible with (2.14) and we conclude that the D5-brane preserves half of the Poincaré supersymmetries.

4. D-branes spanning $AdS_2 \times S^2$

In this section we consider D-branes spanning an $AdS_2 \times S^2$ subspace within $AdS_3 \times S^3$. They can be taken to be embedded at constant $\xi = \xi_0$ and $\psi = \psi_0$ in the coordinate system (2.1).

4.1 D3-branes along $AdS_2 \times S^2$

4.1.1 Near-horizon supersymmetries

We consider a D3-brane probe in this background sitting at constant $\xi = \xi_0$ and $\psi = \psi_0$ and static on M . The worldvolume coordinates can be taken to be $(\tau, \omega, \theta, \phi)$ and we allow for an electromagnetic field on the worldvolume parametrized as

$$2\pi\alpha'F = \tanh \Phi_0 e^0 \wedge e^2 + \tan \Phi_1 e^4 \wedge e^5$$

with $\cos \Phi_1 \geq 0$, e^0 , e^1 as in (3.1) and

$$\begin{aligned} e^4 &= \sqrt{r_1 r_5} \sin \psi_0 d\theta \\ e^5 &= \sqrt{r_1 r_5} \sin \psi_0 \sin \theta d\varphi \end{aligned} \tag{4.1}$$

Then the supersymmetries preserved by the brane are solutions of $(1 - \Gamma)\varepsilon = 0$ where the Killing spinor ε is given in (2.7), (2.8) and

$$\Gamma = e^{-\Phi_0 \Gamma^{02} \sigma^3} e^{-\Phi_1 \Gamma^{45} \sigma^3} \Gamma_{0245} i \sigma^2.$$

Imposing $(1 - \Gamma)\varepsilon = 0$ for all values of $\tau, \omega, \theta, \phi$ leads to four equations

$$\left[1 - e^{-s_2 \Phi_0 \Gamma^{02} \sigma^3} e^{-s_1 \Phi_1 \Gamma^{45} \sigma^3} i \Gamma_{0245} \sigma^2 \right] e^{s_1 \frac{\xi_0}{2} \Gamma^{02} \sigma^1} e^{s_2 \frac{\frac{\pi}{2} - \psi_0}{2} \Gamma^{45} \sigma^1} \varepsilon_0 = 0 \quad (4.2)$$

with $s_{1,2} = \pm 1$. Multiplying with $e^{s_1 \frac{\xi_0}{2} \Gamma^{02} \sigma^1} e^{s_2 \frac{\frac{\pi}{2} - \psi_0}{2} \Gamma^{45} \sigma^1}$ and taking linear combinations one finds that solution exist if

$$\begin{aligned} \tanh \xi_0 &= -\frac{\sin \Phi_1}{\cosh \Phi_0} \\ \cot \psi_0 &= -\frac{\sinh \Phi_0}{\cos \Phi_1} \end{aligned} \quad (4.3)$$

which implies that the worldvolume fluxes take the values

$$\begin{aligned} \tanh \Phi_0 &= -\frac{\cos \psi_0}{\cosh \xi_0} \\ \tan \Phi_1 &= -\frac{\sinh \xi_0}{\sin \psi_0} \end{aligned}$$

and the preserved supersymmetries have to satisfy

$$(1 - \Gamma_{0245} i \sigma^2) \varepsilon_0 = 0. \quad (4.4)$$

Plugging in our ansatz for ε_0 (2.11), this can be written in terms of the 6-dimensional spinor doublets ϵ^\pm as

$$(1 - \gamma_{0245} i \sigma^2) \epsilon^\pm = 0. \quad (4.5)$$

Hence such D3-branes are half-BPS.

In the presence of electric and magnetic worldvolume flux the D3 brane sources the electric NS and R two forms $B^{(2)}$ and $C^{(2)}$ and carries induced F - and D -string charges. This imposes two charge quantization conditions which can be computed from requiring that the solution provides sources for $B^{(2)}$ and $C^{(2)}$ with quantized coefficients. The quantization conditions read:

$$\begin{aligned} \psi_0 &= \frac{g\pi\alpha'}{r_5^2} q = \frac{q}{Q_5} \pi \\ \sinh \xi_0 \sin \psi_0 &= \frac{\pi\alpha'}{r_1 r_5} p \end{aligned} \quad (4.6)$$

where q, p are the induced F - and D -string charges respectively and we have used (2.2). Since $0 \leq \psi_0 \leq \pi$ we see that the radius of the S^2 can take on Q_5 different values. All these solutions preserve the same set of supersymmetries as follows from (4.5). Since the S^2 in S^3 is contractible, these branes do not carry a net $D3$ charge and should be most

likely interpreted as bound states of (p, q) strings ‘puffed-up’ through a version of the Myers effect [28].

The S-dual version of this solution in the F1-NS5 background was constructed and shown to be supersymmetric in [16]. Note that our solution confirms the charge quantization conditions found there which are more subtle in the S-dual background due to the presence of background NS flux.

4.1.2 Poincaré supersymmetries

We can again check whether the solution preserves any Poincaré supersymmetries. For a D3-brane spanning an $AdS_2 \times S^2$ subspace $\xi = \xi_0, \psi = \psi_0$ in global coordinates, the κ -projector in Poincaré coordinates takes the form

$$\Gamma = e^{-\Phi_0 \Gamma_{01} \sigma_3} e^{-\Phi_1 \Gamma_{45} \sigma_3} \Gamma_{\underline{0145}} i \sigma_2.$$

with $\Gamma_{\underline{01}}$ defined in (3.4). Requiring $(1 - \Gamma)\varepsilon_P = 0$ with ε_P given in (2.13), (2.14) for all values of θ, φ on S^2 leads to two equations

$$\left(1 - e^{-s\Phi_0 \Gamma_{\underline{01}} \sigma_3} e^{-\Phi_1 \Gamma_{45} \sigma_3} \Gamma_{\underline{0145}} i \sigma_2\right) e^{s \frac{\xi_0 - \psi_0}{2} \Gamma_{45} \sigma_1} \varepsilon_- = 0 \quad (4.7)$$

with $s = \pm 1$. Multiplying by $e^{s \frac{\xi_0 - \psi_0}{2} \Gamma_{45} \sigma_1}$ and using (2.14) and the equations of motion (4.3) one gets a single condition

$$(1 - \Gamma_{0245} i \sigma_2) \varepsilon_- = 0.$$

This is consistent with (2.14) and again we see that half of the Poincaré supersymmetries are preserved.

4.2 D5-branes spanning $AdS_2 \times S^2$ and wrapping a 2-cycle in M

Next, we consider D5-branes that span and $AdS_2 \times S^2$ subspace and wrap a 2-cycle Σ in M . Although one can construct solutions to the DBI equations of this form, none of them is actually supersymmetric as we will presently show.

Parametrizing the worldvolume flux as

$$2\pi\alpha' F = \tanh \Phi_0 e^0 \wedge e^2 + \tan \Phi_2 e^4 \wedge e^5 + \tan \Phi_2 \text{vol}_\Sigma$$

with e^0, e^2, e^4, e^5 as in (3.1), (4.1) the κ projector is given by

$$\Gamma = e^{-\Phi_0 \Gamma^{02} \sigma_3} e^{-\Phi_1 \Gamma^{45} \sigma_3} e^{-\Phi_2 \Gamma_\Sigma \sigma_3} \Gamma_{024567} \sigma_1$$

with Γ_Σ as in (3.5). Requiring $(1 - \Gamma)\varepsilon = 0$ everywhere leads to four equations

$$\left(1 - s_1 s_2 e^{-s_2 \Phi_0 \Gamma^{02} \sigma_3 - s_1 \Phi_1 \Gamma^{45} \sigma_3} e^{-s_1 s_2 \Phi_2 \Gamma_\Sigma \sigma_3} \Gamma_{0245} \Gamma_\Sigma \sigma_1\right) e^{s_1 \frac{\xi_0}{2} \Gamma^{02} \sigma_1 + s_2 \frac{\pi/2 - \psi_0}{2} \Gamma^{45} \sigma_1} \varepsilon_0 = 0$$

with $s_{1,2} = \pm 1$. Multiplying by $e^{-s_1 \frac{\xi_0}{2} \Gamma^{02} \sigma_1 - s_2 \frac{\pi/2 - \psi_0}{2} \Gamma^{45} \sigma_1}$ $e^{s_2 \Phi_0 \Gamma^{02} \sigma_3 + s_1 \Phi_1 \Gamma^{45} \sigma_3 + s_1 s_2 \Phi_2 \Gamma_\Sigma \sigma_3}$ the four equations can be written out schematically as

$$A(s_1, s_2) \varepsilon_0 = (B(s_1, s_2) \sigma_1 + C(s_1, s_2) i \sigma_2 + D(s_1, s_2) \sigma_3) \varepsilon_0$$

where the coefficients A, B, C, D don't depend on the σ -matrices. Anticommuting the $s_1 = 1, s_2 = 1$ and $s_1 = 1, s_2 = -1$ equations leads to

$$(1 + \sin^2 \psi_0 \cosh 2\Phi_0 \cos 2\Phi_2 + \cos^2 \psi_0 \cos 2\Phi_1) \varepsilon_0 = -\sin^2 \psi_0 \sinh 2\Phi_0 \sin 2\Phi_2 \Gamma^{02} \Gamma_{\Sigma} \varepsilon_0$$

Since $(\Gamma^{02} \Gamma_{\Sigma})^2 = -1$, solutions are possible if both sides vanish separately. In particular, one needs either $\sin \psi_0$, Φ_0 or $\sin 2\Phi_2$ to vanish. We found none of these cases to be consistent with the remaining equations.

4.3 D7-branes spanning $AdS_2 \times S^2 \times M$

4.3.1 Near-horizon supersymmetries

Here we consider a D7-brane spanning and $AdS_2 \times S^2$ subspace in $AdS_3 \times S^3$ at $\xi = \xi_0$, $\psi = \psi_0$ and wrapping the whole of M . Choosing complex coordinates on M , the worldvolume flux can be brought in the form

$$2\pi\alpha' F = \tanh \Phi_0 e^0 \wedge e^2 + \tan \Phi_2 e^4 \wedge e^5 + i \tan \Phi_2 e^1 \wedge e^{\bar{1}} + i \tan \Phi_3 e^2 \wedge e^{\bar{2}}$$

and the κ projector is given by

$$\Gamma = e^{-\Phi_0 \Gamma^{02} \sigma_3} e^{-\Phi_1 \Gamma^{45} \sigma_3} e^{-i\Phi_2 \Gamma^{1\bar{1}} \sigma_3} e^{-\Phi_3 \Gamma^{2\bar{2}} \sigma_3} \Gamma_{0245} \Gamma_{(4)} i\sigma_2.$$

Requiring $(1 - \Gamma)\varepsilon = 0$ and using the chirality property (2.8) leads to four equations

$$(1 + e^{-s_2 \Phi_0 \Gamma^{02} \sigma_3} e^{-s_1 \Phi_1 \Gamma^{45} \sigma_3} e^{-is_1 s_2 \Phi \Gamma^{1\bar{1}} \sigma_3} \Gamma_{0245} i\sigma_2) e^{s_1 \frac{\xi_0}{2} \Gamma^{02} \sigma_1} e^{s_2 \frac{\pi/2 - \psi_0}{2} \Gamma^{45} \sigma_1} \varepsilon_0 = 0 \quad (4.8)$$

with $s_1, s_2 = \pm 1$. Note that only the sum $\Phi \equiv \Phi_2 + \Phi_3$ enters the equations while the difference is unconstrained. Even though solutions of the D-brane Born-Infeld equations exist for general Φ , manipulations similar to the ones in the previous section show that that the above supersymmetry conditions are consistent only for

$$\Phi = 0.$$

Note that this implies that the worldvolume field strength on M is anti-selfdual. In this case, the equations (4.8) reduce (up to a sign difference) to the ones solved in section 4.1. When $M = T^4$, this was to be expected from T-duality, which leaves the background invariant and relates the probe solutions. The solution is given by

$$\begin{aligned} \tanh \xi_0 &= \frac{\sin \Phi_1}{\cosh \Phi_0} \\ \cot \psi_0 &= -\frac{\sinh \Phi_0}{\cos \Phi_1} \\ (1 - \gamma_{0245} i\sigma^2) \varepsilon^{\pm} &= 0. \end{aligned} \quad (4.9)$$

Comparing with (4.5), we note that the S^2 -wrapping D3-branes and D7-branes are mutually BPS.

As before, the values of ξ_0, ψ_0 are quantized in terms of the induced charges carried by the brane. For nonzero Φ_0, Φ_1 , the D7-brane provides a source for the NSNS 2-form $B^{(2)}$

and the D5-brane RR potential $C^{(6)}$ and carries induced F1- and D5- charge. This leads to quantization conditions

$$\begin{aligned}\psi_0 &= \frac{q}{Q_1}\pi \\ \sinh \xi_0 \sin \psi_0 &= \frac{\pi \alpha'}{r_1 r_5} p_5\end{aligned}\tag{4.10}$$

where q and p_5 denote the induced F1- and D5- charge respectively. We see that, in this case, the S^2 radius can take on Q_1 different values, and the corresponding solutions preserve the same set of supersymmetries.

4.3.2 Poincaré supersymmetries

A calculation almost identical to paragraph 4.1.2 shows that these branes preserve half of the Poincaré supersymmetries as well.

5. Discussion and outlook

In this paper, we have constructed a variety of supersymmetric probe brane solutions in the near-horizon D1-D5 background. They are all static with respect to global time and preserve half of the near-horizon supersymmetries. Since the global time generator corresponds to $L_0 + \bar{L}_0$ in the dual CFT, we expect these branes to correspond to supersymmetric conformal operators in the dual CFT. In appendix B we consider branes spanning some other submanifolds, none of which is found to be supersymmetric.

As was mentioned in the Introduction, one of the motivations for studying the branes constructed in this paper is the fact that they share some properties with brane probes in other D-brane backgrounds that have been related to microstates [2, 6–8]. An important open question is therefore whether some of the D-branes considered here can be related to the microstates of the D1-D5 system. One way to clarify their role would be to study their interpretation from the point of view of the dual CFT description of the D1-D5 system (see [10] for a review). Branes spanning an AdS_2 subspace run off to the boundary of AdS_3 where they form a line defect in the dual CFT [30, 22]. Similar AdS_2 branes in the $AdS_5 \times S^5$ background were given a dual CFT interpretation in [21]. We leave this interesting topic for further study. A related issue concerns the relation, if any, of the probe brane solutions considered here and the microstate geometries for the D1-D5 system [29].

It would also be of interest to extend these solutions to the full asymptotically flat geometry. It may be mentioned that in the context of two dimensional black holes, branes with similar properties (in particular in the asymptotically flat geometry) have been discussed [31, 32].

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A. Supersymmetries of the asymptotically flat background

In this appendix we sketch how the Poincaré supersymmetries (2.13), (2.14) arise from the Killing spinors of the full asymptotically flat geometry. We will use the solution for the D1/D5 system given in e.g. [10] The dilatino equation

$$(\Gamma_M \nabla^M \Phi + \Gamma^{\text{MNP}} F_{\text{MNP}}^{(3)} \sigma^1) \varepsilon = 0$$

becomes

$$\left(-\frac{r_1^2}{f_1 r^3} + \frac{r_5^2}{f_5 r^3} \right) \Gamma^2 \varepsilon - \left(\frac{r_1^2}{f_1 r^3} \Gamma^{012} + \frac{r_5^2}{f_5 r^3} \Gamma^{345} \right) \sigma^1 \varepsilon = 0$$

In order to have a solution, we need to impose two projection conditions

$$\Gamma^{01} \sigma^1 \varepsilon = -\varepsilon; \quad \Gamma^{2345} \sigma^1 \varepsilon = \varepsilon \tag{A.1}$$

The second equation can be traded for $\Gamma_{(6)} \varepsilon = -\varepsilon$ and taking the near-horizon limit we recover the two projection conditions (2.14), (2.4) imposed on the Poincaré supersymmetries.

In the near horizon limit however, the first two terms in the above dilatino equation cancel leaving behind a single projection condition equivalent to

$$\Gamma_{(6)} \varepsilon = -\varepsilon \tag{A.2}$$

which becomes the projection condition (2.4) on the near-horizon supersymmetries.

B. D-branes spanning other submanifolds

In the main text, we have considered D-branes which span an AdS_2 subspace in AdS_3 . In this appendix we explore some other possibilities. We restrict attention to D-branes which are static with respect to global time. We find that none of the branes considered here preserve any supersymmetry.

B.1 D3-brane wrapping S^3

Consider a static D3-brane at $\xi = \xi_0$, $\omega = \omega_0$ in AdS_3 and wrapping the S^3 . The world-volume gauge field can be brought in the form

$$2\pi\alpha' F = \tanh \Phi_0 e^0 \wedge e^3 + \tan \Phi_2 e^4 \wedge e^5.$$

One easily checks that the DBI equations of motion impose $\Phi_0 = \xi_0 = \omega_0 = 0$. The conditions for such a solution to preserve supersymmetry are

$$(1 - s_1 e^{-s_1 s_2 \Phi_1 \Gamma^{45} \sigma_3 \Gamma_{0345} i \sigma_2}) \varepsilon_0 = 0$$

for $s_1, s_2 = \pm 1$. One easily checks that the resulting four equations cannot be solved simultaneously; hence there are no supersymmetric solutions in this case.

B.2 Branes spanning AdS_2 wrapping a T^2 within S^3

In this section, we will discuss branes wrapping a $T^2 \in S^3$. One motivation to consider such branes (apart from an exercise of imagination) is from the viewpoint of the R-symmetry of the SCFT dual to the AdS_3 string theory. Extended objects in S^3 will transform non-trivially under the $SO(4)$ isometry of S^3 which becomes the R-symmetry of the Higgs branch. The Coulomb branch has a different R-symmetry. It is interesting to consider the supersymmetry properties of such toroidal branes in S^3 then because such branes do not transform in some obvious manner under $SO(4)$ (in contrast to branes which wrap an S^2 which are conjugacy classes of $SU(2)$ and hence invariant under a vectorial $SU(2)$ of $SO(4)$).

In order to study these branes, we will use Euler-angle co-ordinates on S^3 - the metric takes the form

$$d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2 \tag{B.1}$$

The spin-connection one form can be taken to be

$$\omega_{46} = \cos \theta d\phi_1 \quad \omega_{56} = -\sin \theta d\phi_2$$

with all other components vanishing. Solving the gravitino equations the sphere part of the killing spinor

$$e^{\frac{\theta}{2}\Gamma_{45}\sigma^1} e^{-\frac{\phi_1}{2}\Gamma_{46}} e^{-\frac{\phi_2}{2}\Gamma_{46}\sigma^1} \varepsilon_0$$

with ε_0 a constant spinor. Note that the $4 = \phi_1, 5 = \phi_2, \theta = 6$.

The κ symmetry matrix is

$$\Gamma = e^{-\frac{\xi}{2}(\Phi_0\Gamma_{02} + \Phi_1\Gamma_{45})} \Gamma_{0245} i\sigma^2$$

As before, we demand that $\Gamma\varepsilon = \varepsilon$ for all $\tau, \omega, \phi_1, \phi_2$ and its a simple matter to check that this gives equations which cannot be satisfied (so long as ξ_0 is finite i.e., the brane is at a finite radius away from the boundary). More specifically, the incompatible equations are those which come from imposing $\Gamma\Gamma_{45}\varepsilon = \Gamma_{45}\varepsilon$ and $\Gamma\Gamma_{45}\sigma^1\varepsilon = \Gamma_{45}\sigma^1\varepsilon$ (the latter two are the conditions which come from requiring that the kappa symmetry condition hold for all $\phi_{1,2}$).

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